Journal of Engineering Physics and Thermophysics, Vol. 73, No. 3, 2000

WEAK EVAPORATION (CONDENSATION) WITH AN ARBITRARY EVAPORATION COEFFICIENT IN GASES WITH A CONSTANT FREQUENCY OF MOLECULAR COLLISIONS

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UDC 533.72

We have obtained an exact solution of the problem on determination of temperature and concentration jumps of a rarefied elementary (monoatomic) gas in the case of weak evaporation from the gas–condensed phase interface into the half-space of a saturated vapor. Use is made of a model kinetic equation with a collision integral in the BGK (Bhatnagar, Gross, and Krook) form. Accurate coefficients of the temperature and concentration jumps are found. It is shown that at low evaporation coefficients the dependence of the concentration of the vapor on its evaporation (concentration) rate at a large distance from the interface is linear even at low (as compared to sonic) vapor velocities.

The problem on weak evaporation has attracted attention for several decades. This is associated with the theoretical significance of the problem and numerous practical applications. This problem has been investigated with use of both the analytical methods [1-4] for model kinetic equations and the approximate and numerical methods for a complete Boltzmann equation [5-8]. In all the mentioned works, the case where the evaporation coefficient is equal to unity has been studied. An exact solution with the use of the model kinetic equations was obtained comparatively recently in our works [1, 4].

The importance of allowing for the evaporation coefficient is related, in particular, also to the fact that according to experimental data [9] it can vary within wide limits from 0.006 to 1. The evaporation coefficient was taken into consideration in the works devoted to strong evaporation (condensation) when the Mach number is about unity and in connection with heterogeneous chemical reactions [10-14]. In these works, the possibility of different regimes depending on the accommodation coefficient was not studied; particular emphasis was placed on obtaining numerical and approximate solutions.

In the present work, an analytical solution of the half-spatial boundary-value problem on weak evaporation (condensation) is obtained for the first time for the case where the evaporation coefficient acquires arbitrary values from 0 to 1. Weak evaporation (condensation) occurs at a rate of $U > 0$ ($U < 0$) from a plane gas–condensed phase interface into the half-space of a saturated vapor. Use is made of the model kinetic equation with the collision integral in the BGK form. Separation of the variables leads to a characteristic equation, the eigenvectors of which are in the space of generalized functions. It is established that the initial boundary-value problem is expanded in terms of these vectors. A proof of the expansion reduces to a solution of the vector boundary-value Riemann–Hilbert problem [15]. At first the corresponding homogeneous boundary-value problem is solved. At this point, the so-called fundamental matrix function is constructed, which is used for solving an inhomogeneous problem. The general solution of the latter is found in the class of meromorphic functions. From the solvability conditions, we find the coefficients of discrete and continuous spectra of the expansion of the solution of the boundary-value problem, including exact formulas for calculation of a temperature jump and dependences of the vapor concentration near the wall and at a large distance from it on the evaporation coefficient and on the evaporation or condensation rate. From the formulas obtained it is seen that at small values of the evaporation coefficient the dependence of the vapor concentration on the rate U is nonlinear even at small (as compared to sonic) rates of vapor evaporation (condensation).

Moscow Pedagogical University, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 73, No. 3, pp. 542-549, May–June, 2000. Original article submitted December 4, 1998; revision submitted April 13, 1999.

Let us introduce the Cartesian coordinate system with the center on the gas–condensed phase interface and the x axis directed perpendicularly to the interface in the direction of the gas. At the interface, the processes of evaporation or condensation occur, as a result of which there exists the mean mass velocity U of the gas (vapor) in the direction from or to the interface. This velocity is considered to be much lower than the sonic one, thus allowing linearization of the kinetic equation.

In the general case, the problem of boundary conditions is rather complicated [16]. As the wall condition, we take

$$f^+(0, \mathbf{v}) = \alpha f_s(T_s, v) + (1-\alpha) f_0(T_s, v), \quad v_x > 0. \tag{1}$$

Here, α is the evaporation coefficient, i.e., the amount of molecules incident onto the wall which are condensed on it; $1-\alpha$ is, respectively, the amount of molecules incident onto the wall which are reflected from it. In this approach, the evaporation and condensation coefficients are assumed to be equal. In condition (1):

$$f_s(T_s, v) = n_s \left(\frac{m}{2\pi k T_s}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_s}\right),$$

$$f_0(T_s, v) = \frac{n_0}{n_s} f_s(T_s, v)$$

(n_s is the saturated-vapor concentration at temperature T_s, $n_0 = n(0)$ is the concentration of the vapor molecules on the wall (more exactly, in the immediate vicinity of it)). The quantity n_0 is determined from the tightness condition (wall impermeability) for the reflected molecules

$$(1-\alpha) \int v_x [f_0(T_s, v)\, \theta_+(v_x) + f^-(0, \mathbf{v})\, \theta_+(-v_x)]\, d^3v = 0, \tag{2}$$

where $\theta_+(x)$ is the Heaviside function: $\theta_+(x) = 1$, $x > 0$; $\theta_+(x) = 0$, $x < 0$.

We seek the distribution function in the form

$$f = f_i(T_s, v)(1 + \varphi), \quad f_i = \frac{n_i}{n_s} f_s, \tag{3}$$

where $n_i = n(\infty)$ is the concentration of the gas molecules at infinity to be determined.

Substituting (3) into (1) we obtain an equation from which we find

$$\varphi^+(0, \mathbf{c}) = \alpha \frac{n_s - n_0}{n_i} + \frac{n_0 - n_i}{n_i}, \quad c_x > 0. \tag{4}$$

The quantities n_0 and n_i are unknown and must be determined from a solution of the kinetic equation. Next, we will find the relation between n_0 and n_i from the condition of equality of mass flows on the wall and at a large distance from it. For this, we use the law of mass flow conservation

$$\pi^{-3/2} \int \exp(-c^2)\, c_x \varphi(x, \mathbf{c})\, d^3c = C_0 = \text{const}. \tag{5}$$

Substituting the asymptotics of the distribution function at a large distance from the wall into this equality

$$\varphi_{as}(x, \mathbf{c}) = 2Uc_x + \varepsilon_t (c^2 - 3/2), \tag{6}$$

we find that

$$C_0 = U\,. \tag{7}$$

Here, $\varepsilon_t = T_i/T_s - 1$ is the sought temperature jump, $T_i = T(\infty)$.

Now we use the impermeability condition (2) for the reflected molecules. In this equality we calculate the integral of the first term

$$\int v_x f_0\,(T_s, v)\,\theta_+\,(v_x)\,d^3v = \frac{n_0}{2\sqrt{\pi\beta_s}}\,,\quad \beta_s = \frac{m}{2kT_s}\,;$$

consequently, passing to a dimensionless molecular velocity, we obtain

$$\frac{n_0}{2\sqrt{\pi}} = -\int f^{-}\,(0, \mathbf{c})\,c_x\,\theta_+\,(-c_x)\,d^3c\,,$$

whence with the use of (1), (4), (5), and (7) we have

$$\frac{n_s}{n_i} - \frac{n_0}{n_i} = 2\sqrt{\pi}\,\frac{U}{\alpha}\,. \tag{8}$$

Thus, the problem consists of solving the BGK equation [1-4]

$$c_x\frac{\partial\varphi}{\partial x} + \varphi\,(x, \mathbf{c}) = \pi^{-3/2}\int \exp\,(-c^2)\,k\,(\mathbf{c}, \mathbf{c}')\,\varphi\,(x, \mathbf{c}')\,d^3c' \tag{9}$$

with the boundary conditions

$$\begin{gathered}\varphi\,(0, \mathbf{c}) = \varepsilon_n\,,\quad c_x > 0\,;\\ \varphi\,(x, \mathbf{c}) = \varphi_{as}\,(x, \mathbf{c}) + o\,(1)\,,\quad x \to +\infty\,,\quad c_x < 0\,,\end{gathered} \tag{10}$$

where the function φ_{as} is determined by equality (6) and ε_n is the right-hand side of condition (4).

If we seek a solution of problem (9), (10) in the form

$$\varphi = \varepsilon_n + h_1\,(x, \mu) + (c^2 - 3/2)\,h_2\,(x, \mu)\,,\quad \mu = c_x\,,$$

we obtain the vector boundary-value problem for the column vector $h = [h_1\ h_2]^t$ (t is the transposition):

$$\mu\frac{\partial h}{\partial x} + h\,(x, \mu) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} K_0\,(\mu, \mu')\,h\,(x, \mu')\,d\mu'\,, \tag{11}$$

$$h\,(0, \mu) = \mathbf{0}\,,\quad \mu > 0\,, \tag{12}$$

$$h\,(x, \mu) = h_{as}\,(x, \mu) + o\,(1)\,,\quad x \to \infty\,,\quad \mu < 0\,; \tag{13}$$

$$h_{as}\,(x, \mu) = \begin{vmatrix} 2U\mu - \varepsilon_n \\ \varepsilon_t \end{vmatrix}\,,$$

$$K_0\,(\mu, \mu') = \left[E + 2\mu\mu'\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\right] K\,(\mu)\,,$$

$$K(\mu) = \begin{vmatrix} 1 & l(\mu) \\ \frac{2}{3} l(\mu) & \frac{2}{3} l(\mu)^2 + \frac{2}{3} \end{vmatrix},$$

here E is the unit matrix, $\mathbf{0} = [0\ 0]^t$ is the null vector, and $l(\mu) = \mu^2 - 1/2$.

Separation of variables $h_\eta(x, \mu) = \exp(-x/\eta)F(\eta, \mu)$ allows Eq. (11) to be reduced to the characteristic one

$$(\eta - \mu) F(\eta, \mu) = \frac{1}{\sqrt{\pi}} \eta E$$

with unit normalization

$$\int_{-\infty}^{\infty} \exp(-\mu^2) K(\mu) F(\eta, \mu)\, d\mu = E.$$

From the characteristic equation with allowance for the normalization at $\eta \in (-\infty, +\infty)$ we find the eigenvectors of the continuous spectrum in the space of generalized functions [17]

$$F(\eta, \mu) = \frac{1}{\sqrt{\pi}} P \frac{1}{\eta - \mu} E + \exp(\eta^2) B(\eta)\, \delta(\eta - \mu).$$

Here Px^{-1} is the distribution, i.e., the main value of the integral of x^{-1}, $\delta(x)$ is the Dirac delta function, $B(\eta) = K^{-1}(\eta)\Lambda(\eta)$, $\Lambda(z)$ is the dispersion matrix, and $\Lambda(z) = \lambda(z)K(z) + P(z)$; here

$$P(z) = \begin{vmatrix} 0 & 1/2 \\ 1/3 & 1/3\,(z^2 + 1/2) \end{vmatrix},$$

$$\lambda(z) = 1 + z \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-\tau^2)}{\tau - z}\, d\tau.$$

We will establish that a solution of problem (11)-(12) can be represented in the form of expansion in terms of the eigenvectors

$$h(x, \mu) = h_{as}(\mu) + \int_0^{\infty} \exp(-x/\eta) F(\eta, \mu)\, a(\eta)\, d\eta, \tag{14}$$

where $a(\eta)$ is the coefficient of the continuous spectrum, i.e., the unknown vector function

$$h_{as}(\mu) = \begin{vmatrix} 2U\mu - \varepsilon_n \\ \varepsilon_t \end{vmatrix}.$$

Using boundary conditions (12) and (13), we pass from expansion (14) to the singular integral equation with the Cauchy kernel

$$h_{as}(\mu) + \frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{\eta a(\eta)}{\eta - \mu} d\eta + \exp(\mu^2) B(\mu) a(\mu) = \mathbf{0}, \quad \mu > 0. \tag{15}$$

Introduce the unknown vector function

$$N(z) = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{\eta a(\eta)}{\eta - z} d\eta \tag{16}$$

and reduce Eq. (15) to the inhomogeneous vector boundary-value Riemann–Hilbert problem

$$B^+(\mu)\,[N^+(\mu) + h_{as}(\mu)] = B^-(\mu)\,[N^-(\mu) + h_{as}(\mu)]\,, \quad \mu > 0\,, \tag{17}$$

with the matrix coefficient $G(\mu) = [\Lambda^+(\mu)]^{-1}\Lambda^-(\mu) = [B^+(\mu)]^{-1}B^-(\mu)$. At first we will consider the corresponding homogeneous boundary-value problem (the problem of coefficient factorization)

$$G(\mu) = X^+(\mu)\,[X^-(\mu)]^{-1}\,, \quad \mu > 0\,, \tag{18}$$

where $X(z)$ is the unknown matrix function that is analytical in the complex plane with a branch cut along the positive semiaxis unlike the matrix $G(z)$ that is piecewise-analytical in the upper and lower half-planes with a branch cut along the entire real axis.

We seek a solution of problem (18) in the form $X(z) = S(z)U(z)S^{-1}(z)$, where $S(z)$ is the matrix leading to the diagonal form of the matrix $B(z)$; $U(z) = \text{diag}\,\{U_1(z)), U_2(z\}$ is the new unknown diagonal matrix. From the definition of the diagonalizing matrix it follows that

$$S^{-1}(z)\,B(z)\,S(z) = \Omega(z) = \text{diag}\left\{\Omega_1(z), \Omega_2(z)\right\},$$

where

$$\Omega_j(z) = \lambda(z) + \frac{1}{4}\left[-z^2 + \frac{3}{2} + (-1)^j r(z)\right], \quad j = 1, 2\,;$$

$$r(z) = \sqrt{q(z)}\,, \quad q(z) = \left(z^2 - \frac{3}{2}\right)^2 + 4\,.$$

Substituting $X(z)$ into (18), we arrive at the matrix boundary-value problem equivalent to two scalar problems:

$$U_j^+(\mu) = \exp(-2i\theta_j(\mu))\,U_j^-(\mu)\,, \quad j = 1, 2\,, \quad \mu > 0\,, \tag{19}$$

where $\theta_j(\mu) = \arg \Omega_j^+(\mu)$ is the regular branch of the argument of the function $\Omega_j^+(\mu)$ fixed by the condition $\theta_j(0) = 0$, $j = 1, 2$.

We note that the matrix function $X(z)$ is analytical in the complex plane with the branch cuts Γ_1 and Γ_2 connecting the points of branching, i.e., nulls of the polynomial $q(z)$ in the upper and lower half-planes. Thus, for the matrix function $X(z)$ to be single-valued, the condition $X^+(\tau) = X^-(\tau)$, $\tau \in \Gamma$, $\Gamma = \Gamma_1 \cup \Gamma_2$, must be fulfilled on these branch cuts or

$$U^+(\tau)\,[S^+(\tau)]^{-1}\,S^-(\tau) = [S^+(\tau)]^{-1}\,S^-(\tau)\,, \quad \tau \in \Gamma\,.$$

Taking into consideration that

$$[S^+(\tau)]^{-1}\,S^-(\tau) = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix},$$

from the previous condition we obtain two more problems which, unlike scalar problems (19), are vector boundary-value ones:

$$U_1^+(\tau) = U_2^-(\tau), \quad U_1^-(\tau) = U_2^+(\tau), \quad \tau \in \Gamma. \tag{20}$$

Thus, it is necessary to construct a solution that would simultaneously satisfy the scalar problems (19) on the main branch cut and vector boundary-value problems (20) on additional branch cuts. The methods for solving such problems, which are also encountered in the theory of radiation transfer (see, e.g., [18]), were developed in [1-4]. Therefore, we give a solution of problems (19) and (20) without derivation:

$$U_j(z) = \exp\left[-A(z) + (-1)^j r(z)\,(B(z) - R(z))\right], \quad j = 1, 2.$$

Here

$$A(z) = \frac{1}{2\pi}\int_0^\infty \left[\theta_1(\tau) + \theta_2(\tau) - 2\pi\right]\frac{d\tau}{\tau - z},$$

$$R(z) = \int_0^{x_0} \frac{d\tau}{r(\tau)\,(\tau - z)},$$

$$B(z) = \frac{1}{2\pi}\int_0^\infty \frac{\theta_1(\tau) - \theta_2(\tau)}{r(\tau)}\frac{d\tau}{\tau - z};$$

the point x_0 is found from the Jacobi inversion problem for elliptic integrals

$$\frac{1}{2\pi}\int_0^\infty \frac{\theta_1(\tau) - \theta_2(\tau)}{r(\tau)}d\tau + \int_0^{x_0}\frac{d\tau}{r(\tau)} = 0.$$

Thus, the matrix $X(z)$ is constructed. With the aid of (18) we transform (17) to the problem of determination of an analytic vector function by its zero step

$$[X^+(\mu)]^{-1}\,[N^+(\mu) + h_{as}(\mu)] = [X^-(\mu)]^{-1}\,[N^-(\mu) + h_{as}(\mu)], \quad \mu > 0,$$

the general solution of which has the form

$$N(z) = -h_{as}(z) + X(z)\,\Phi(z); \tag{21}$$

where $\Phi(z)$ is the column vector with elements

$$\Phi_1(z) = \alpha_1 z + \alpha_0 + \frac{\alpha_{-1}}{z - x_0}, \quad \Phi_2(z) = \beta_1 z + \beta_0 + \frac{\beta_{-1}}{z - x_0};$$

here α_i and β_i ($i = -1, 0.1$) are arbitrary constants.

Based on (16) and (21) and using the Sokhotskii formula, we will find the coefficient of the continuous spectrum

$$2\sqrt{\pi}\,i\eta a(\eta) = [X^+(\eta) - X^-(\eta)]\,\Phi(\eta).$$

All free parameters of solution (21) are determined uniquely from the conditions of elimination of its special features. Determination of these parameters completes the proof of expansion (14). As a result of determination of all of the free parameters of solution (21), we arrive at a formula for the dependence of the temperature jump on the evaporation (condensation) rate U

$$\varepsilon_t = 2U\gamma_t, \quad \gamma_t = 4x_0 \exp(-2B_2 - 2R_2) D,$$

and also at the dependence of the concentration on U:

$$\alpha \frac{n_s - n_0}{n_i} - \frac{n_0 - n_i}{n_i} = -2U\gamma_n, \quad \gamma_n = -A_1 + B_3 + R_3 + x_0(1 + 8D) - \gamma_t. \tag{22}$$

Here

$$D = \frac{3x_0^2/2 - r^2(0) - r(0)\, r(x_0)}{r(x_0)\,[r(x_0) + x_0^2 + r(0)]^2},$$

$$A_1 = -\frac{1}{2\pi} \int_0^\infty [\theta_1(\tau) + \theta_2(\tau) - 2\pi]\, d\tau,$$

$$B_k = -\frac{1}{2\pi} \int_0^\infty \frac{\theta_1(\tau) - \theta_2(\tau)}{r(\tau)} \tau^{k-1}\, d\tau,$$

$$R_k = -\int_0^{x_0} \frac{\tau^{k-1}}{r(\tau)} d\tau, \quad k = 2, 3.$$

Numerical calculations show that $\gamma_t = -0.22436$ and $\gamma_n = -0.84350$. It is reasonable to compare these accurate values of the coefficients with the approximate ones from [7]: $\gamma_t = -0.223375$ and $\gamma_n = -0.842645$.

Now from Eqs. (8) and (22) we find that

$$\frac{n_i}{n_s} = \frac{\alpha}{\alpha + 2U[\sqrt{\pi} + \alpha(\sqrt{\pi} + \gamma_n)]},$$

$$\frac{n_0}{n_s} = \frac{\alpha[1 + 2U(\sqrt{\pi} + \gamma_n)]}{\alpha + 2U[\sqrt{\pi} + \gamma_n]}. \tag{23}$$

We consider two limiting cases:

(1) $U \ll \alpha$; here

$$\frac{n_0}{n_s} = \frac{n_i}{n_s} = 1 - \frac{2U}{\alpha}[\sqrt{\pi} + \alpha(\sqrt{\pi} + \gamma_n)], \tag{24}$$

i.e., the vapor concentration n_i at a large distance from the surface slightly differs from the saturated-vapor concentration n_s; consequently, here a purely linear (with respect to the evaporation/condensation rate) variant of the problem is realized;

(2) $\alpha \ll U$; here

$$\frac{n_i}{n_s} = \frac{1}{2\sqrt{\pi}} \frac{\alpha}{U}, \quad \frac{n_0}{n_s} = \alpha\left[1 + \frac{\gamma_n}{\sqrt{\pi}} + \frac{1}{2\sqrt{\pi}\, U}\right], \tag{25}$$

i.e., the vapor concentration at a large distance from the surface tends to zero with the evaporation coefficient α tending to zero, since at low values of the evaporation coefficient the gas outflow from the surface (proportional to the rate U) is not compensated by the inflow of vapor molecules due to evaporation.

From formula (24) it follows that in case (1) the processes of evaporation and condensation are symmetrical. At the same time, in case (2), as is seen from formula (15), a limitingly asymmetrical variant of the process, which occurs only in evaporation, is realized. Indeed, for the condensation process the rate U is negative. From (25) it follows that in this case the molecular concentration at a large distance from the wall must be negative.

We note that the dependence of the vapor concentration on the evaporation or condensation rate at a large distance from the interface is, generally speaking, nonlinear (see formula (23)). This does not contradict the use of the linearized collision integral since under the conditions where the evaporation or condensation rate is much lower than the sonic one, a correction for the equilibrium distribution function f_0 remains small. However, in this case, the equilibrium distribution function f_i at infinity (at a large distance from the wall) can considerably differ from the equilibrium distribution function f_s corresponding to the saturated-vapor concentration, i.e., n_i can be much smaller than n_s.

This work was carried out with financial support from the Russian Fund for Fundamental Research (project 99-01-00336).

NOTATION

f_0, equilibrium distribution function; f_s, distribution function of the saturated vapor; f_i, distribution function of the vapor at a large distance from the wall; m, molecular mass; $\mathbf{v}$, dimensional velocity of the molecules; $\mathbf{c}$, dimensionless velocity of the molecules, $\mathbf{c} = \sqrt{m/2kT_c}\,\mathbf{v}$; k, Boltzmann constant; α, evaporation coefficient; T_s, temperature of the saturated vapor; n_s, concentration of the saturated vapor; n, vapor concentration near the wall; n_i, vapor concentration at a large distance from the wall. Subscripts: s, surface; i, infinity; as, asymptotics; t, temperature; n, concentration.

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